Minimizing Interference in Unmanaged Environments of Densely Deployed Wireless Access Points Using a Graphical Game Model

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***Abstract*—Urban residential areas are becoming increasingly dense with more and more wireless home networks being deployed in close proximity. Considering that in such a dense urban residential area, each unit has its own wireless access point (AP), deployed without any coordination with other such units, then a need arises for reducing interference and increasing overall Quality of Experience (QoE) of the clients involved. To do this, we propose that *neighbouring* APs — i.e., APs that are physically close to each other — form groups, where one member of the group serves the terminals of all group members in addition to its own terminals, while the other APs of the group can be silent or even turned off. The fact that participating units are deployed without any coordination makes the overall QoE vulnerable to the selfish behaviour of each unit. We propose a *cooperative-neighbourhood* graphical game model comprising of a network of selfishly-oriented nodes represented by a graph where the outgoing links of a certain node capture the improvement in utility that a neighbour's client may experience from a potential cooperation. We show and prove that using the proposed model provides motivation for APs to enter and remain in cooperative neighbourhoods, in which interference is decreased due to the voluntary cooperation of the neighbours.**

I. Introduction

The paper considers a dense urban residential area where each home unit has its own IEEE 802.11 based wireless access point (AP), deployed without any coordination with other such units. Lacking any management regarding the efficient utilization of the communication channels, it is quite common for a terminal served by one of the APs to be within the signal range of multiple alternative APs. Since APs can be in competition for the same communication resource (radio channel), and the current standards dictate that at any given time every terminal must be rigidly associated with one particular AP, this situation results in increased interference and consequently a low utilization efficiency of the radio resource, when same or overlapping channels are selected by neighbouring APs.

In a dense deployment, it would be much better for individual APs that are in physical proximity to each other to form groups, where one member of the group would serve the terminals of all group members in addition to its own terminals, so that the other access points of the group can be silent or even turned off, thereby reducing interference and increasing overall *Quality of Experience* (QoE). In this study, these groups include only members whose signal strength is sufficient to serve all group members. It is reasonable to assume that all APs have an active own client that they serve when active whether they are in cooperation with a neighbour or not.

Since there is no centralized entity that can control the APs and force them to form cooperative groups, the creation of such groups must be able to arise from a distributed process where each AP makes its own decisions independently and rationally for the benefit of itself and its terminals. *Graphical game theory* [21] is an appropriate tool to model such decentralized, topology-dependent schemes.

The basic motivation behind the proposed approach, is the inefficiency of wireless communication caused by interference when multiple closely located APs, using the same or overlapping channels operate at the same time. If the clients of all APs are served by only one AP (even if these alternate from time to time), the interference between them is avoided and they receive service much closer to the theoretical limit of the radio resource. In this paper, we model the idea of cooperative neighbourhhoods as a graphical game and show that there exists motivation towards the cooperation of individual units, where a home unit is represented in the model by a dual nature node, a node encapturing both a server (AP) and a client. The strategical decision of such a unit to voluntarily participate in a group where members serve clients of neighbouring nodes in addition to their own, has the property that a unit is more likely to gain more in terms of QoE, than a unit defecting from such cooperation. We will henceforth refer to the proposed graphical game model as the *cooperative-neighbourhood game*.

The paper is organized as folows. Section II overviews related work; specifically, section II-A discusses wireless deployment in urban environments whereas section II-B describes the use of game theoretic approaches in distributed situations and section II-C gets into some more detail on graphical games. Section III describes the situation scenario for which the proposed model is considered, and section IV presents the graphical game model in a given neighbourhood. Section V illustrates the motivation of cooperation through a targeted usage example of the proposed model, leading to an interesting Nash Equilibrium for two players in Section VI. Finally, Section VII offers some conclusions and future work directions.

II. Related Work

A. Wireless deployments in urban environments

Density of wireless networks in residential areas is on the rise with more and more home networks being deployed in close proximity, enabled by the low cost and easy deployment of off-the-shelf IEEE 802.11 hardware and other personal wireless technologies. It is not uncommon for a wireless station to be within range of dozens of APs [2], competing for the limited number of channels offered by the IEEE 802.11 wireless standard, unlike organizations and campuses where experts can carefully control and manage interference by planning the setup of the network in advance [3]. Wireless networks in residential environments have a number of characteristics that make their deployment more challenging. For instance, the network is unplanned, thus aspects of planning such as coverage and interference cannot be controlled. Furthermore, deployments are mostly spontaneous, resulting in uneven density of deployment, the network lacks aspects such as efficient placement of access points, troubleshooting and adapting to network changes such as traffic load, as well as security issues. The authors of [2] use the term chaotic deployments to refer to such a collection of wireless networks which are unplanned and unmanaged. However, they do mention advantages of such chaotic networks, for instance easily enabling new techniques to determine location [4] or providing near ubiquitous wireless connectivity [5]. The main disadvantage of these chaotic deployments is that interference can significantly affect end-user performance, while being hard to detect [6]. In this paper we consider a solution based on "virtualization" among the interfering APs, where APs serve each others' clients. The security implications of allowing association of clients across APs from multiple owners are being addressed, e.g. in [7]. In this paper we focus on the *incentive* aspect of a particular model of cooperation through the use of graphical game theoretic tools, and propose a framework to ensure that the APs are indeed motivated to provide service to each others' clients.

B. Strategical Decision-Making

In this paper, we consider the interactions between the individual units in dense urban deployments of wireless networks, represented by dual nature vertices in a graph. Describing and analysing interactions between independent, *selfish* entities is a situation that makes a good candidate to be modeled using the theoretical framework of Game Theory. Game Theory provides appropriate models and tools to handle multiple, interacting entities attempting to make a decision, and seeking a solution state that maximizes each entity's utility, i.e. each entity's *quantified satisfaction*. Game Theory has been extensively used in networking research as a theoretical decision-making framework, e.g. for routing [8], [9], congestion control [10], [11], resource sharing [12], [13], and heterogeneous networks [14], [15].

In [16] the authors address cooperative neighbourhoods by concentrating on the *Prisoner's Dilemma/Iterated Prisoner's Dilemma* game model and proposing a group strategy to motivate adjacent neighbours into cooperation. The Prisoner's Dilemma and Iterated Prisoner's Dilemma [17] have been a rich source of research since the 1950s. The publication of Axelrod's book in 1984 [18] was the main driver that boosted the concept to the attention of areas outside game theory, as a model for promoting cooperation. The empirical results of the Iterated Prisoner's Dilemma (IPD) tournaments organized by Axelrod have influenced the game theory, machine learning and evolutionary computation communities, showing how features such as adaptivity and group play can result in gains at indiviual level. In fact, adaptive players, learning from the games in which they are involved, are more likely to survive than non-adaptive players in evolutionary IPD [19] and group strategies performed extremely well and defeated well-known strategies in round-robin competitions in the 2004 and 2005 IPD tournaments [20]. In this paper, the idea of cooperative neighbourhoods is revisited by using the idea of a graph to set the neighbourhood map and defining a graph game where cooperation can result in gains for the individual nodes.

C. Graphical Games

Graphical games are one-shot games that model multi-player interaction in situations where restrictions and influences among the player population may exist. In fact, graphical games are a more efficient representation of one-shot multi-player games that cannot efficiently be represented with the normal form representation of a game (where rows and columns in a table represent the available actions and resulting payoffs of the game players). In addition to providing a better representation for multi-player games, graphical games are meant to capture locality and how the game is affected by the player's positioning. As such, we use graphical games in this paper for modelling and analyzing a multi-entity problem characterized by complex interactions, from the communication networking field [21].

Graphical games adopt a simple graph-theoretic model, where a game is represented by a graph G in which players are identified with vertices. A player of vertex i has payoffs that are entirely specified by the actions of i and those of its neighbour set in G, i.e. the set of vertices thar have a direct link to i. Thus the graph of a game defines structural constraints over players' strategic influences towards other players. To fully describe the graphical game, in addition to the graph itself, the numerical payoff functions to each player must be specified.

III. The scenario

Currently, dense residential deployments of home wireless networks consist of uncoordinated APs that serve their terminals individually. The APs do not form groups and share the communication channel, which is an unmanaged common resource, resulting in a low utilization efficiency due to the competition between the APs and the interference it causes. This interference can be reduced if the APs can form groups according to their location, such that any APs belonging to the same group can serve any terminal associated with any of the other group members.

It is possible for an AP to recognize its *neighbourhood* from the signals it receives, having a knowledge of the required signal strength thresholds that would serve its terminals in a satisfying manner, i.e. with the required perceived QoE. In such a neighbourhood only one of the APs needs to assume the role of a leader, while the others can remain silent, and thereby minimize the interference and improve the overall QoE for all terminals involved. The role of the leader can be assumed on a rotating basis. Of course, in order to take part in a cooperative neighbourhood, the APs need to be motivated to act cooperatively, i.e. have an incentive to be silent or turned off while it is the turn of another AP to serve, and to serve everyone's terminals once their own turn comes. We show how such a distributed logic can be motivated and sustained in the neighbourhood using a graphical game model where each AP can make an independent decision whether or not to cooperate in such manner with its neighbours.

The interactions in a cooperative neighbourhood can be modelled as a game between the participating units, represented as vertices on a graph that captures through its links the signal received at each node from the neighbouring nodes. Given that each member of the neighbourhood has two choices at any given time: (a) to cooperate with one or more neighbours or (b) serve only its own client, the graphical game model captures the utility of each node according to the decisions made by the whole neighborhood, including itself, and provides a framework for scheduling the activation and deactivation times for the servers (APs) of cooperating nodes[1]. Which of the two behaviours to select in each round depends on the strategy of behaviour that a player has decided to follow during the game. The strategy of each *player*, i.e. of each unit, is selected such that it results in the highest possible payoff for the particular player, regardless of the strategies selected by the other neighbouring nodes. We refer to such interaction as a *cooperative-neighbourhood game*.

[1]Note that units may be a part of more than one neighbourhood, i.e. receive a good signal from peers that are in different neighbourhoods, making the scheduling task more challenging. This is part of future work.

IV. The graphical game model of a cooperative neighbourhood

The paper uses graphical game theory tools to define a network of APs represented using graph theory. The relationship between any two neighbouring APs is represented in terms of the QoE that an AP's client perceives, if served by the neighbouring AP. In other words, the *satisfaction* that the client of an AP perceives, when the client is served by a particular neighbouring AP and all other signals are off (which may not always be the case), is captured by the proposed model. To capture and further quantify this *satisfaction*, the model represents each AP as a node on a graph and includes weights on the incoming links of each node, showing the perceived QoE based on the signal(s) received at the node.

A. The graph

We consider a set of nodes $V = \{v_1, v_2, v_3, ...v_n\}$ located on a plane. Each node is considered as an entity comprising of one server and a constant number of clients. Without loss of generality we assume that each server serves one client, and consequently each node employs dual functionality, i.e. both the functionality of a server and the functionality of a client. Therefore, a server node serves its client node, using broadcast transmission. At a time t, we say that node v_i is active or *ON*, if its server is broadcasting (to its own client). Otherwise, we say that the node is inactive or *OFF*.

Consider two nodes, v_i, v_j, $i \neq j$, which are close enough to detect each other's signals. In particular, node v_i may receive information from node v_j when it is *OFF* while node v_j is *ON*, i.e. when node v_j broadcasts. If the quality of the signal received by node v_i from node v_j is above some lower bound value assumed by the node v_i, we may consider that there exists a directed edge from node v_j to node v_i, denoted as (v_j, v_i). Moreover, we can quantify the quality of the received signal by having a weight $w(v_j, v_i)$ associated with the directed edge (v_j, v_i). Normalizing this value we consider that $w(v_j, v_i) \in (0, 1)$. Finally, this value is analogous to the quality of the received signal, i.e. better received quality corresponds to value of $w(v_j, v_i)$ close to 1. Summing up, this reasoning motivates the following mathematical definition for the two nodes:

Definition 1. Consider two nodes v_i and v_j such that the two nodes can detect each other's signals and the node v_i can receive information from node v_j when it is OFF while node v_j is ON, of quality above some lower bound value. Then, we assume that there exist a directed edge from node v_j to node v_i, denoted as (v_j, v_i) of a positive weight $w(v_j, v_i) \in (0, 1)$. The weight is analogous to the quality of the received signal at node v_i.

Thus, more formally, using the set of nodes V, we define a weighted graph $G = (V, \overrightarrow{E}, \overrightarrow{W})$, where $\overrightarrow{W} : \overrightarrow{E} \to [0, 1]$ and $(v_i, v_j) \in \overrightarrow{E}$, $i \neq j$, if and only if node v_j can receive the signal of node v_i when v_i is *ON* and v_j is *OFF*. The measurement of the quality of the received signal by node v_i's client once it is *OFF* and cooperates with node v_j, which is *ON*, using edge (v_i, v_j), is given by weight $w(v_i, v_j)$. Thus, $w(v_i, v_j)$ is positive if and only if $(v_i, v_j) \in \overrightarrow{E}$. We assume that if $w(v_i, v_j) > 0$, then $w(v_j, v_i) > 0$ for all nodes $v_i, v_j \in V$, $i \neq j$.

B. The Time

We consider a basic unit of time period T, e.g. 1 hour, and we split the time period T into x smaller time periods or slots $T_1, T_2, \ldots T_x$ such that for each $T_k \in T$ there exists at least one node that in a group of colocated nodes that alternates between the *ON* and *OFF* mode, and remains *ON* for the whole time slot T_k. So, $\bigcup_{T_k} = T$ and $T_k \bigcap T_l = \emptyset, k \neq l$, i.e. the sum of the time slots is the time period T and no two time slots overlap. By $|T_k|$ we denote the time elapsed from the beginning of time slot T_k until the end of the time slot.

Fix a time slot T_k. Then, for any node $v_i \in V$,

$$Mode(T_k, v_i) = \begin{cases} 0, & \text{if node } v_i \text{ is } \textit{OFF} \text{ in time slot } T_k \\ 1, & \text{if node } v_i \text{ is } \textit{ON} \text{ in time slot } T_k \; . \end{cases} \tag{1}$$

So, for node v_i the time period T can be partitioned into two sets $ON_T(v_i)$ and $OFF_T(v_i)$, where $ON_T(v_i) = \{T_k \mid Mode(T_k, v_i) = 1\}$ and $OFF_T(v_i) = \{T_k | Mode(T_k, v_i) = 0\}$.

C. Cooperative and Non-Cooperative Neighbours

Within a given time period T, node v_i may be in *agreement* or in *cooperation* with some of its neighbouring nodes. For any node v_j, being in agreement with node v_i, $i \neq j$ means that node v_j broadcasts only when v_i does not broadcast and serves the client of node v_i in addition to its own client, during the time that node v_i is *OFF*. The set of the neighbours of node v_i that are in agreement during time T, is denoted by $Coop_T(v_i)$, while the complete set of neighbours of node v_i is denoted by $Nei(v_i)$, where $Coop_T(v_i) \subseteq Nei(v_i)$ and $Mode(T_k, v_i) = 1 - Mode(T_k, v_j)$ for each $v_j \in Coop_T(v_i)$. So, $ON_T(v_i) = OFF_T(v_j)$ and $ON_T(v_j) = OFF_T(v_i)$ for each $v_j \in Coop_T(v_i)$. On the other hand, the set of neighbours with which v_i is *not* in agreement with is denoted as $NCoop_T(v_i)$, where $NCoop_T(v_i) \subseteq Nei(v_i)$ and it may be that $Mode(T_k, v_i) = Mode(T_k, v_j)$ where $v_j \in NCoop_T(v_i)$.

D. Interference and Quality of Experience

1) Received Signal: We are interested in measuring the QoE received by the client of node v_i through the signal strength broadcasted by v_i, at any time slot T_k during the time period T. The received QoE, considering both cooperative and non-cooperative neighbours during time period T, is approximated, for simplicity, as the summation of the edge weights of all interfering nodes. We denote this quantity as $rS_{T_k}(v_i)$. We also distinguish two kinds of signals received at node v_i:

- the signal received by its client when node v_i is *ON* denoted as $ONrS_{T_k}(v_i)$, and
- the signal received by its client when node v_i is *OFF* denoted as $OFFrS_{T_k}(v_i)$.

At any time T_k the node is either *ON* or *OFF*. Thus,

$$rS_{T_k}(v_i) = \begin{cases} OFFrS_{T_k}(v_i), & \text{if node } v_i \text{ is } \textit{OFF} \\ ONrS_{T_k}(v_i), & \text{if node } v_i \text{ is } \textit{ON} \; . \end{cases} \tag{2}$$

2) ON operation: Fix a time slot $T_k \in T$. When node v_i is *ON* and none of its neighbours broadcast at the same time, the client of node v_i receives the broadcast signal in the best quality (no interference) and hence experiences the best possible QoE. We assume that top QoE is measured by a unit. Therefore, we set the experienced QoE by the node's client functionality to be equal to 1 in this case.

When node v_i is *ON*, none of its neighbours in cooperation, i.e. set $Coop_T(v_i)$ are *ON* at the same time. However, some of its neighbours not in cooperation may be *ON*, i.e. set $NCoop_T(v_i)$. In the second case, interference occurs and the signal received by node v_i's client is degraded causing a degraded QoE. We consider the degradation to be analogous to the strength of the signal of neighbour v_j received at node v_i and is captured by the weight $w(v_j, v_i)$ of edge (v_j, v_i). This degradation is also increased as more than one non-cooperative neighbours broadcast at the same time as node v_i. For simplicity, we consider the degradation to be analogous to the strengths of their signals received at node v_i. Thus,

$$ONrS_{T_k}(v_i) = max\{0, 1 - \sum_{v_j \in Nei_{T_k}(v_i)} w(v_j, v_i) \cdot Mode(T_k, v_j)\} \tag{3}$$

3) OFF operation: On the other hand, when node v_i is *OFF*, the quality of the signal received at node v_i's client, and hence the QoE experienced, depends on the number of neighbours in cooperation with node v_i, that are *ON* at time T_k and serving the clients of the nodes in cooperation that are *OFF*, including the client of node v_i. If there exists one such neighbouring node v_j that is *ON*, the quality of the signal is captured by the weight $w(v_j, v_i)$ of edge (v_j, v_i). However, if there exist more than one neighbouring nodes not in cooperation with node v_i that are *ON* at the same time, this results in interference received at node v_i, which degrades the experienced QoE at the client of node v_i. This paper considers the QoE to be analogous to the strengths of the received signals.

We assume that node v_i is tuned to the strongest signal of its neighbours in set $Coop_T(v_i)$. Note that the quality of this signal is also degraded by the sum of the received signals from cooperative and non-cooperative neighbours that are *ON* at the same time as node v_i is. Thus,

$$OFFrS_{T_k}(v_i) = max\left\{0, (w(maxCoop_\sigma(T_k, i), v_i) - \sum_{\substack{v_h \in Nei(v_i) \\ v_h \neq maxCoop_\sigma(T_k, i)}} \right\} \tag{4}$$

where $maxCoop_\sigma(T_k, i) \in V$, such that $Mode(T_k, maxCoop_\sigma(T_k, 1)) = 1$ and $w(maxCoop_\sigma(T_k, i), i) = max_{j \in Coop_\sigma(i)}\{w(j, i) \cdot Mode(T_k, j)\}$

So, summing up for the time period T:

$$\begin{aligned} rS_T(v_i) &= \sum_{T_k \in T | Mode(T_k, v_i) = ON} ONrS_{T_k}(v_i) \cdot |T_k| \\ &+ \sum_{T_k \in T | Mode(T_k, v_i) = OFF} OFFrS_{T_k}(v_i) \cdot |T_k| \end{aligned}$$

E. The Cooperative Neighbourhood Game

We consider an one-shot strategic game resulting from the described scenario in which the players of the game are the nodes (servers). Given the decisions of the nodes whether to operate in *ON* or *OFF* operation during each time slot $T_k \in T$, the utility of player v_i is the received signal of v_i's client during time period T, given by $rS_T(v_i)$.

Thus, more formally we define:

Definition 2. The game $\Gamma(V, \overrightarrow{E}, \overrightarrow{W})$ is defined as follows. The set of players of the game is the set V. For simplicity, we define $V = \{1, \ldots, n\}$. A profile σ of the game is associated with the basic time period T of the scenario described. T is split into time slots $T_1, T_2, \ldots T_x$, such that $\bigcup_{T_k} = T$ and $T_k \bigcap T_l = \emptyset, k \neq l$, T_k corresponding to the smallest time slot where we may have alterations between *ON* and *OFF* operations of the nodes.

The strategies of the players in a profile σ are defined as follows:

The strategy of player (node) i is given by $\sigma_i = (Mode_\sigma(T), Coop_\sigma(i))$, where $Mode_\sigma(T)$, a vector of 0s and 1s, such that $Mode_\sigma(T_k, i) = 0$ *or* 1, based on whether node i operates in *ON* or *OFF* mode during time slot T_k, for each $T_k \in T$. $Coop_\sigma(v_i) \subseteq Nei_\sigma(v_i)$ is the set of neighbouring nodes of node i, with which node i has decided to cooperate with in σ. Cooperation means that for each such cooperative neighbour j of node i, $Mode_\sigma(T_k, i) = 1 - Mode_\sigma(T_k, j)$ for each $T_k \in T$, and the two nodes are in agreement to serve each other's client.

For player (node) i denote, $maxCoop_\sigma(T_k, i) \in V$, such that $Mode(T_k, maxCoop_\sigma(T_k, 1)) = 1$ and $w(maxCoop_\sigma(T_k, i), i) = max_{j \in Coop_\sigma(i)}\{w(j, i) \cdot Mode(T_k, j)\}$

Then, the utility of player (node) i, representing the signal received as expressed in equations (2), (3), (4), is given by:

$$U_\sigma(i) = \sum_{T_k \in ON_T(i)} ONU_{T_k}(i) \cdot |T_k| + \sum_{T_k \in OFFU_T(i)} OFF_{T_k}(i) \cdot |T_k| \quad (5)$$

where,

$$ONU_{T_k}(i) = max\{0, (1 - \sum_{j \in Nei(i)} w(j, i) \cdot Mode(T_k, j))\} \quad (6)$$

and,

$$OFFU_{T_k}(i) = max\{0, (w(maxCoop_\sigma(T_k, i), i) - \sum_{\substack{h \in Nei(i) \\ h \notin maxCoop_\sigma(T_k, i)}} \quad (7)$$

V. A Usage Example

In this section, we consider a simple scenario to demonstrate the interaction between two interacting neighbours, nodes 1 and 2. We juxtapose the situation where the two neighbours do not cooperate versus the case where the two neighbours cooperate, and we look for the time split between *ON* and *OFF* time slots for which cooperation is beneficial. We assume that the signal strength received by a node, from the node's neighbour, is more than half of the normalized maximum, i.e. $w(1, 2),\ w(2, 1) > 0.5$. In particular, we assume that $w(1, 2) = 0.6$ and $w(2, 1) = 0.7$. We consider three case studies for the two nodes, where in the first case the two nodes do not cooperate, in the second case they cooperate and the time split between *ON* and *OFF* times is such that the *ON* time for one of the nodes is much larger than its *OFF* time, and in the third case, we select equal *ON* and *OFF* times. The motivation behind the usage example is to determine the role of a particular time split towards the decision of the nodes to cooperate. We discover that in both cases there exists a motivation to cooperate, which leads to the generalization of these findings for the cooperation of two nodes in the subsequent section.

1) Case 1: No Cooperation: Let us first consider the case of the two nodes not in cooperation. Each node is *ON*, with its own client receiving the maximum signal from its own server, i.e. 1, while simultaneously the adjacent node is *ON* causing a continuous interference. Hence according to equation (5),

$$\begin{aligned} U_\sigma(1) &= \sum_{T_k \in ON_T(1)} ONU_{T_k}(1) \cdot |T_k| \\ &+ \sum_{T_k \in OFF_T(1)} OFFU_{T_k}(1) \cdot |T_k| \end{aligned}$$

Since $ON_T(1) = 1 = ON_T(2)$ and $OFF_T(2) = OFF_T(1) = \emptyset$. Thus by equations 6 and 7, it follows,

$$\begin{aligned} U_\sigma(1) &= \sum_{T_k \in T} ONU_{T_k}(1) \cdot |T_k| \\ &+ \sum_{T_k \in \{\emptyset\}} OFFU_{T_k}(1) \cdot |T_k| \\ &= \sum_{T_k \in T} ONU_{T_k}(1) \cdot |T_k| + \emptyset \\ &= ON_T(1) \cdot |T| \end{aligned}$$

where,

$$\begin{aligned} ONU_{T_k}(1) &= max\{0, (1 - \sum_{j \in \{2\}} w(j, 1) \cdot Mode(T_k, j))\} \\ &= max\{0, 1 - w(2, 1) \cdot 1\} \end{aligned}$$

Thus,

$$\begin{aligned} U_\sigma(1) &= max\{0, (1 - w(2,1))\} \cdot |T| \\ &= max\{0, 1 - w(2,1)\} \cdot |T| \\ &= 1 - 0.7 \cdot 1 = 0.3 \end{aligned} \tag{8}$$

Similarly, for node 2, we get that,

$$\begin{aligned} U_\sigma(2) &= ON_T(2) \cdot |T| \\ &= max\{0, (1 - w(2,1))\} \cdot |T| \\ &= 1 - 0.6 \cdot 1 = 0.4 \end{aligned} \tag{9}$$

2) *Case 2: Time Split T_1 = 0.8, T_2 = 0.2:* Consider now the case that the two nodes are in cooperation, then they alternate between states of *ON* and *OFF* times, while each node serves the client of its neighbour during its *ON* time, while its client is being served by the neighbour's server, during its *OFF* time. Thus, $|T| = |T_1| + |T_2|$ and $T_1 \bigcap T_2 = \{\emptyset\}$. Here we assume that $T_1 = 0.2$, $T_2 = 0.8$.

$$ON_T(1) = \{T_1\} = OFF_T(2),\ OFF_T(1) = \{T_2\} = OFF_T(2)$$

Therefore,

$$\begin{aligned} U_\sigma(1) &= \sum_{T_k \in ON_T(1)} ONU_{T_k}(1) \cdot |T_k| \\ &\quad + \sum_{T_k \in OFF_T(1)} OFFU_{T_k}(1) \cdot |T_k| \\ &= \sum_{T_k \in \{T_1\}} ONU_{T_k}(1) \cdot |T_k| \\ &\quad + \sum_{T_k \in \{T_2\}} OFFU_{T_k}(1) \cdot |T_k| \\ &= ONU_{T_1}(1) \cdot |T_1| + OFFU_{T_2}(1) \cdot |T_2| \end{aligned} \tag{10}$$

where,

$$\begin{aligned} ONU_{T_1}(1) &= max\{0, (1 - \sum_{i \in \{2\}} w(i,1) \cdot Mode(T_1, i))\} \\ &= max\{0, 1 - w(2,1) \cdot \emptyset\} = 1. \end{aligned}$$

Also, $maxCoop_\sigma(T_1, 1) = 2$. Thus,

$$\begin{aligned} OFFU_{T_2}(1) &= max\left\{0, w(2,1) \right. \\ &\qquad \left. - \sum_{\substack{h \in \{2\} \\ h \neq 2}} w(h,1) \cdot Mode(T_2, h) \right\} \\ &= max\{0, w(2,1) - \emptyset\} = 0.7. \end{aligned}$$

Thus,

$$\begin{aligned} U_\sigma(1) &= 1 \cdot |T_1| + w(2,1) \cdot |T_2| \\ &= 1 \cdot 0.8 + 0.7 \cdot 0.2 = 0.94 \end{aligned} \tag{11}$$

Similarly, for node 2 we get that:

$$\begin{aligned} U_\sigma(2) &= 1 \cdot |T_2| + w(1,2) \cdot |T_1| \\ &= 1 \cdot 0.2 + 0.6 \cdot 0.8 = 0.68. \end{aligned} \tag{12}$$

3) *Case 3: Time Split T_1 = 0.5, T_2 = 0.5:* In this section, we consider a similar case scenario with two interacting nodes 1 and 2 in cooperation but we modify the proposed time split of the total time period T, to be such that $T = T_1 + T_2$, $T_1 \bigcap T_2 = \{\emptyset\}$ and $T_1 = 0.5$, $T_2 = 0.5$. Thus, $ON_T(1) = \{T_1\} = OFF_T(2)$ and $OFF_T(1) = \{T_2\} = OFF_T(2)$. So, by equations (11) and (12),

$$U_\sigma(1) = 1 \cdot 0.5 + 0.7 \cdot 0.5 = 0.85$$

$$U_\sigma(2) = 1 \cdot 0.5 + 0.6 \cdot 0.5 = 0.8$$

Next, we discuss and prove that for weights above the value of 0.5, cooperation is the most profitable option regardless of the particular time split between the *ON* and *OFF* times for the situation of two neighbouring nodes.

VI. Nash Equilibrium for two neighbouring nodes

For ease of exposition, we consider the simplest case where the *cooperative-neighbourhood* game is played between two adjacent nodes 1 and 2, where the signal broadcast by each node's server is received by the other node's client in addition to the broadcasting node's own client. Let the weights on their incident edges denoted by $w(1,2)$ and $w(2,1)$. Remember that the utility of each node is defined in equation (5), and elaborated in equations (6) and (7).

Note that if a node decides to cooperate with at least one neighbour, then it is *OFF* for some of the time. On the other hand, if it does not cooperate with any neighbour at any time, then it is *ON* for the whole time T. In the following we compare values of utility in cooperation and non-cooperation of the two nodes and characterize the strategy that results in a Nash Equilibrium for the two nodes.

A. Two Interacting Neighbouring Nodes

Next, we compare the utilities of node 1 and node 2 in both the case where they cooperate and the case they do not cooperate, and we show a characterization of a cooperative profile in order to be a Nash Equilibrium. We first consider the non-cooperation case and give the utilities of the two nodes resulted as functions of the weights of their edges and then prove necessary conditions to get a Nash equilibrium for the two nodes.

1) *No cooperation of both nodes:* We first consider that both nodes are *ON* for the whole duration of time period T, i.e. they do not cooperate with each other. Let σ be the resulting profile. Then, the utilities of the nodes are given in equations (9), (9):

$$U_\sigma(1) = max\{0, 1 - w(2,1)\} \cdot |T| \tag{13}$$

$$U_\sigma(2) = max\{0, (1 - w(1,2))\} \cdot |T| \tag{14}$$

B. Equilibrium for the two nodes

We now show the necessary conditions to get Nash equilibria as functions of the weights of the edges connecting the two nodes.

Theorem VI.1. Assume two interacting nodes 1, 2 with $w(1,2)$ and $w(2,1)$ the weights of the link between them. Then cooperation when the two nodes split T into any two sets T_1, T_2 such that $T_1 \bigcap T_2 = \emptyset$ and $T = T_1 \bigcup T_2$ is a Nash Equilibrium for the nodes if both $w(1,2)$ and $w(2,1) \geq 0.5$.

Proof: We assume first that the two nodes agreed to cooperate in T and compute their utilities. Assume without loss of generality that the two nodes agreed to split T into two parts, T_1 and T_2, such that $T_1 \bigcap T_2 = \emptyset$ and $T = T_1 \bigcup T_2$ and $Mode(T_1, 1) = 1 = 1 - Mode(T_1, 2)$ and $Mode(T_2, 2) = 1 = 1 - Mode(T_2, 1)$. Let σ be the resulting profile. Then, by the utilities of the nodes are given in equations (11) and (12):

$$U_\sigma(1) = 1 \cdot |T_1| + w(2,1) \cdot |T_2|$$

and

$$U_\sigma(2) = 1 \cdot |T_2| + w(1,2) \cdot |T_1|.$$

We now show that any unilateral alternations of any of the two nodes do not increase their utilities. Each of the two nodes has two possible alternations:

1) To increase its ON period.
2) To decrease its ON period.

We first consider the second option, i.e. for the node to decrease its ON period. Let t be the increased time in which node 1 is OFF. Let σ' be the resulting profile.

Let T_1', T_2' be the new split of T. Consider first node 1. Then, $|T_1|' = |T_1| - |t|, |t| > 0$ and $|T_2|' = |T_2| + |t|$. Note that during time period t, node 1 will receive, by equation 5:

$$\begin{aligned} U_{\sigma'}(1) &= ONU_{T_1'}(1) \cdot |T_1'| + OFFU_{T_2'}(1) \cdot |T_2'| \\ &= = (|T_1| - |t|) + w(2,1) \cdot Mode(T_2, 2) \cdot |T_2| \\ &\quad + w(2,1) \cdot Mode(t, 2) \cdot |t| \\ &= (|T_2| - |t|) + w(2,1) \cdot |T_2| + w(2,1) \cdot 0 \\ &= |T_1| - |t| + w(2,1) \cdot |T_2| \\ &= U_\sigma(1) - |t| \\ &< U_\sigma(1), \end{aligned} \tag{15}$$

by equation (**??**) since $|t| > 0$.

Thus node 1 has no gain if it changes according to the second option. Similarly, we can show that $U_{\sigma'}(2) < U_\sigma(2)$. Therefore, the second option of a node decreasing its ON period does not result in some gain for any of the two nodes 1 or 2.

Next, we consider the first option of a node to increase its ON period. Let T_1', T_2' be the new split of T, $T_1' \bigcup T_2' = T$, $T_1' \bigcap T_2' = \emptyset$. Assume that there exists some $t > 0$, such that without loss of generality, node 1 increases its ON time period to $|T_1|' = |T_1| + |t|$ and $|T_2|' = |T_2| - |t|$. Note that nothing changes for node 2, i.e. $Mode(T_1, 2) = OFF$ and $Mode(T_2, 2) = ON$. Also, $Mode(t, 2) = ON$. Let σ' be the resulting profile.

Then the utility of node 1 in σ' by equation (5) is,

$$\begin{aligned} U_{\sigma'}(1) &= ONU_{T_1'}(1) \cdot |T_1'| + OFFU_{T_2'}(1) \cdot |T_2'| \\ &= 1 \cdot |T_1| + (1 - w(2,1)) \cdot |t| + w(2,1) \cdot (|T_2| - |t|) \\ &= (|T_1| + w(2,1) \cdot |T_2|) + |t| - 2|t| \cdot w(2,1) \\ &= U_\sigma(1) + |t| - 2|t| \cdot w(2,1), \end{aligned} \tag{16}$$

by equation (**??**) since $|t| > 0$.

In order for σ' to be a better response for node 1, it must be that:

$$\begin{aligned} U_{\sigma'}(1) &> U_\sigma(1) \\ U_{\sigma'}(1) - U_\sigma(1) > 0 & \\ U_\sigma(1) + |t| - 2|t| \cdot w(2,1) - U_\sigma(1) &> 0, \text{ by eq. (16)} \\ 2|t| \cdot w(2,1) &< |t| \end{aligned}$$

Therefore,

$$w(2,1) < \frac{1}{2}$$

A contradiction, since $w(2,1) \geq \frac{1}{2}$ by assumption.

Similarly, we can show that for node 2 to benefit from increasing its ON time, it should be that $w(1,2) < \frac{1}{2}$, a contradiction by assumption.

Thus, if any of the nodes 1, 2 unilaterally alters its strategy to the first option of increasing its ON time, then profile σ' does not result to a better response than the strategy in σ. Therefore, σ is a Nash Equilibrium.

The theorem is now complete. ■

VII. Conclusion

The paper investigated the interactions between wireless access points that operate in the same geographical region without any coordination. Using a graphical game theoretic model, we showed that the players are motivated to create alliances with their neighbours so as to serve their terminals jointly and in a coordinated manner, leading to the decrease or even elimination of interference for all cooperating neighbours, and, therefore, a boost to the Quality of Experience observed by the client devices. The theoretical analysis shows the value of the cooperation for the interacting neighbours to form such a cooperative neighbourhood.

The cooperative neighbourhood model is represented as a game on a graph where the outgoing links of each node represent the signal strength (or interference depending on whether the node receiving the signal is *ON* or *OFF*) received from a neighbour, and consequently quantifies the *satisfaction* that may result from cooperation. We have provided a theoretical analysis and characterized the equilibrium between two players interacting in a cooperative neighbourhood, which is a first step towards the study of more general and complex game models and solutions in the future.

Our main goal in this study has been to demonstrate how graphical games are a promising approach for modelling interference situations in chaotic wireless network deployments. To that end, our model and analysis have not looked at some important practical considerations that will need to be addressed in future work before the results can be used in a real cooperation protocol, such as neighbour discovery and synchronization; we also employed a simplified linear Quality of Experience metric that does not capture the full complexities of wireless radio propagation and interference. An extension of particular theoretical interest would be a situation where a wireless access point serves more than one client device, and only some but not all of those devices can be covered by a neighbour. The analysis of equilibria in graphical representations of cooperative neighbourhoods with partial overlapping relationships, and the necessary corresponding extension and alleviation of the constraints on the *ON* and *OFF* times of each player, is a subject of ongoing work.

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